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FUNDAMENTAL SENSITIVITY LIMITS DUE TO PROPAGATION MEDIUM INHOMOGENEITIES

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SUMMARY

Increasing communication range requirements and improvement of oscillator short-term stability focus attention on time-varying medium inhomogeneities induced modulation as a potential fundamental limitation to system sensitivity. This report considers two such media, the earth's atmosphere and the solar corona.

It is concluded that tropospheric fluctuations set a minimum bandwidth limitation of the order of 10^{-5} to 10^{-6} cps which is quite irrelevant at the present or foreseeable state of the art.

Solar corona is found to be a more nearly significant limitation. The effect is worst at low frequencies and depends markedly on the range of closest approach of the line-of-sight to the sun. Sufficient data is not available to directly infer phase modulation effects at approaches closer than 0.3 A.U. to the sun, however, indications of extrapolation laws are given. At 0.3 A.U. and for 1000 mc the total predicted phase modulation is 0.4 rad rms, in a bandwidth of 1/3 cps.

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1. INTRODUCTION

As extraterrestrial probes reach to ever more remote regions of space and with the development of improved very stable oscillators it may be predicted that much narrower band communications links will be needed and will be feasible insofar as equipment limitations are concerned.

A fundamental limitation to the increase in sensitivity improvements available in this direction, however, arises from the random modulations imposed on the signal by the time changing inhomogeneities of the propagation medium.

This report considers two such effects in detail, namely, that due to the earth's lower atmosphere and that due to solar corona which affects rays passing near the sun.

2. TROPOSPHERIC FLUCTUATIONS

Fluctuations due to turbulent inhomogeneities in our own atmosphere are, at least to first order, over the frequency range of interest independent of frequency and thus constitute the principal source of fluctuation for sufficiently high frequencies. Figure 1* summarizes diverse measurements of the spectrum of apparent path length fluctuations due to this source in terms of the normalized spectrum

$$G'(f) = G_{\dot{r}}(f)/L \quad \left(\frac{ft}{sec}\right)^2 \cdot \left(\frac{sec}{ft}\right) \qquad 1)$$

where L is the effective path length in the atmosphere, ft.

For a path through and out of the atmosphere as to a deep-space probe one can take approximately

$$L = (17{,}000 \text{ ft}) \cdot \mathrm{Csc}\, E \qquad 2)$$
$$E = \text{elevation angle}$$
for $E > 5^o$.

It is clear that these curves can all be reasonably well approximated by an expression of the form

$$G'(f) = Kf^{-\alpha} \qquad 3)$$

where, matching over the range

$$10^{-1} < f < 1 \quad \text{(cps)}$$
$$K \approx 10^{-14} \text{ to } 10^{-12} \quad (\text{ft sec}^{\alpha - 1}) \qquad 4)$$
$$\alpha \approx 2 \text{ to } 3$$

The corresponding phase fluctuation spectrum follows from the relation

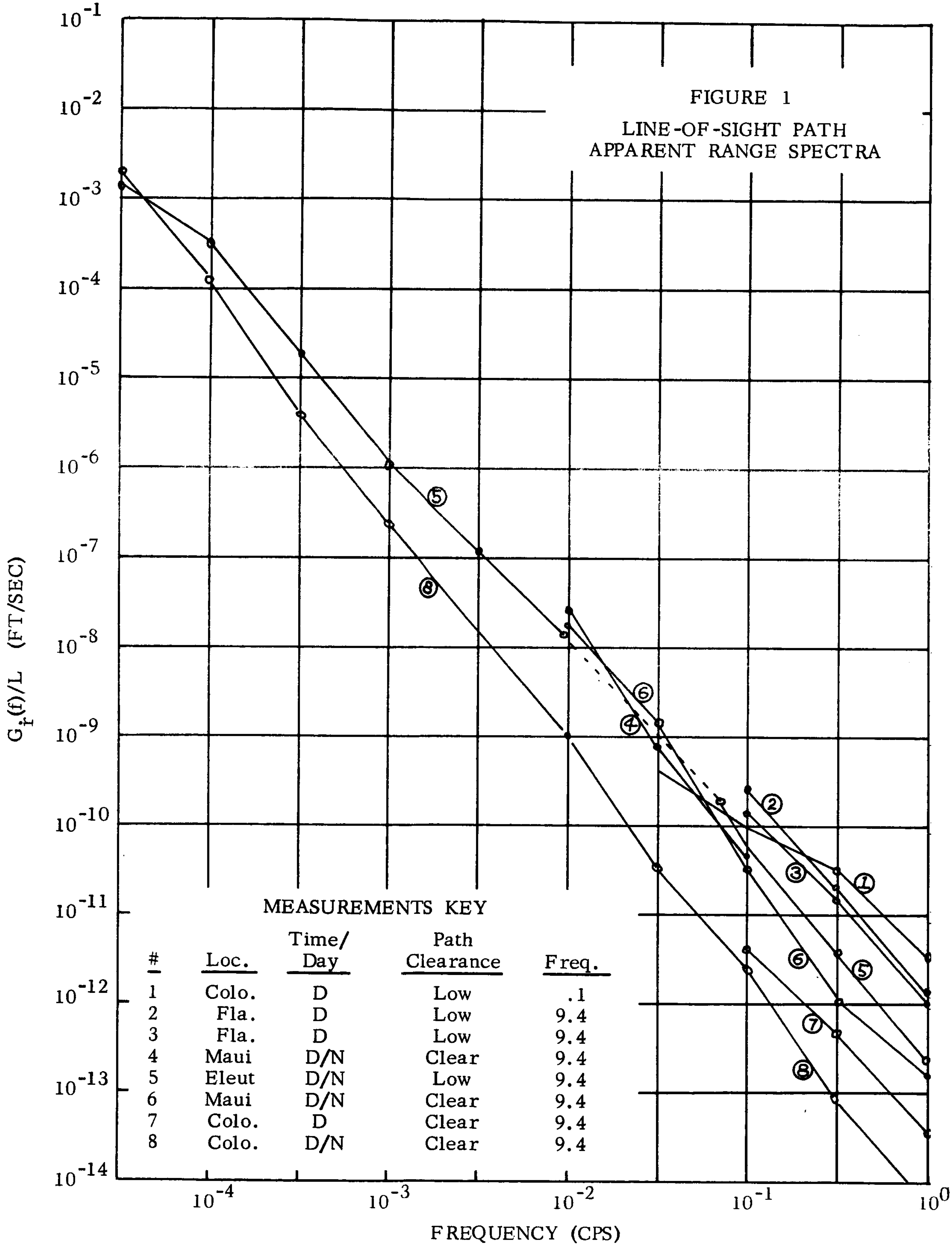
$$\varphi = \frac{2\pi R}{\lambda} \qquad 5)$$

where λ is the carrier wavelength.

Thus

$$G_r(f) = \frac{1}{(2\pi f)^2} G_{\dot{r}}(f) \qquad 6)$$

* See Page 7 for data references



#	Loc.	Time/ Day	Path Clearance	Freq.
1	Colo.	D	Low	.1
2	Fla.	D	Low	9.4
3	Fla.	D	Low	9.4
4	Maui	D/N	Clear	9.4
5	Eleut	D/N	Low	9.4
6	Maui	D/N	Clear	9.4
7	Colo.	D	Clear	9.4
8	Colo.	D/N	Clear	9.4

and

$$G_\varphi(f) = \frac{1}{\lambda^2 f^2} G_{\dot{r}}(f)$$

$$\approx \frac{KL}{\lambda^2} f^{\alpha - 2} \qquad 7)$$

The effect of such an input phase spectrum on a tracking loop is to impose a minimum tracking bandwidth and, therefore, a sensitivity limit. This limitation corresponds roughly to the point at which the untracked phase fluctuation, i.e., the rms phase tracking error is of the order of 1 radian. This is not an abrupt loss-of-lock threshold but a serious loss of sensitivity occurs for much smaller bandwidths, with a corresponding increased probability of loss-of-lock due to noise. If we denote

$$Y(s) = \text{the phase-locked loop error transfer function}$$

$$= \frac{\theta_{error}(s)}{\theta_{input}(s)}$$

where $s = 2\pi i f$

Then

$$\overline{\epsilon^2} = \int_0^\infty G_\varphi(f) \left| Y(s) \right|^2 df \qquad 8)$$

Y(f) depends on the nature of the tracking loop. For a common second-order loop

$$Y_2(s) = \frac{\dfrac{s^2}{\omega_o^2}}{1 + \dfrac{\sqrt{2}}{\omega_o} s + \left(\dfrac{1}{\omega_o^2}\right) s^2} \qquad 9)$$

while a third order loop

$$Y_3(s) = \frac{\dfrac{s^3}{\omega_o^3}}{1 + \dfrac{2}{\omega_o} s + \dfrac{2}{\omega_o^2} s^2 + \dfrac{1}{\omega_o^3} s^3} \qquad 10)$$

The corresponding single-sided noise bandwidths in cps are

$$B_2 = \frac{3}{4\sqrt{2}} \omega_o$$

$$B_3 = \frac{5}{6} \omega_o$$

The resulting integral, 8), is of particularly simple form if we take $\alpha = 2$ in 7) for then, for the second-order case

$$\overline{\epsilon_2^2} = \frac{KL}{\lambda^2} \int_o^\infty f^{-4} \frac{\frac{s^4}{\omega_o^4}}{\left|1 + \frac{\sqrt{2}}{\omega_o} s + \frac{1}{\omega_o^2} s^2\right|^2} df$$

$$= \frac{KL}{\lambda^2} (2\pi)^3 \int_o^\infty \frac{\frac{1}{\omega_o^2} d\omega}{\left|1 + i\frac{\sqrt{2}}{\omega_o} \omega - \frac{1}{\omega_o^2} \omega^2\right|^2} \qquad 11)$$

where $s = i\omega = 2\pi i f$

The integral is easily evaluated by the method of James, Nichols and Phillips (MIT Rad. Lab., V. 25) as $\frac{\pi}{2\sqrt{2}\,\omega_o}$ so that

$$\overline{\epsilon_2^2} = \frac{KL\, 2^{3/2} \pi^4}{\lambda^2 \omega_o}$$

or in terms of the noise bandwidth

$$\boxed{\overline{\epsilon_2^2} = \frac{3\pi^4 KL}{2\lambda^2 B_2}} \qquad 12)$$

Similarly for the third order loop

$$\overline{\epsilon_3^2} = \frac{KL}{\lambda^2} \int_0^{\infty} \frac{f^{-4} \frac{s^6}{\omega_o^6} df}{\left| 1 + \frac{2}{\omega_o} s + \frac{2}{\omega_o^2} s^2 + \frac{1}{\omega_o^3} s^3 \right|^2}$$

$$= \frac{KL (2\pi)^3}{\lambda^2} \int_0^{\infty} \frac{- \frac{\omega^2}{\omega_o^6} d\omega}{\left| 1 + i\frac{2}{\omega_o} \omega - \frac{2}{\omega_o^2} \omega^2 - \frac{i}{\omega_o^3} \omega^3 \right|}$$

$$= \frac{KL \pi (2\pi)^3}{10\lambda^2 \omega_o}$$

$$\boxed{\overline{\epsilon_3^2} = \frac{2}{3} \frac{\pi^4 KL}{\lambda^2 B_3}} \qquad 13)$$

Note that the second and third order cases are very nearly the same differing only in a small constant.

To evaluate these results consider the second order case, taking

$$\sqrt{\overline{\epsilon_2^2}} = 1 \text{ rad}$$

$$K = 10^{-13}$$

$$L = 17{,}000 \text{ Csc } 5^o$$

$$= 195{,}000 \text{ ft}$$

$$\lambda = .45 \text{ ft} \quad \text{(S-Band)}$$

Then solving for B_3

$$B_3 = \frac{2}{3} \frac{\pi^4 \, 10^{-13} \, 195{,}000}{(.45)^2}$$

$$= 6.25 \cdot 10^{-6} \text{ cps} \qquad 14)$$

This is the minimum tracking bandwidth in order that tropospheric phase fluctuations not adversely affect loop track reliability. It is clear that fluctuations from tropospheric inhomogeneities will never be a problem in terms of bandwidth limitations as compared to other sources of fluctuation such as oscillator jitter.

Curve #	Reference For Fig. 1
1,2,3,7	"Final Report on Phase Stability Studies of Ground-to-Ground Microwave Link", NBS Report 6702, 14 June,1960.
4	Norton, et al,"An Experimental Study of Phase Variations in Line-of-Sight Microwave Transmissions", NBS Monograph 33, 1 Nov.,1961.
5	Janes, et al, "Phase and Amplitude Diversity in Over-Water Transmissions at Two Microwave Frequencies", NBS Report No. 7656, 26 Feb.,1963.
6	Janes and Thompson,"Preliminary Report on Experimental Studies of Atmospheric Errors in Microwave Tracking Systems",NBS Report 8237, 4 March,1964.
8	Thompson, Janes, and Kirkpatrick,"An Analysis of Time Variations in Tropospheric Refractive Index and Apparent Path Length", Jour.Geophy. Res.,V.65,No.1, p.193, January,1960.

3. SOLAR CORONA

Since about 1951 numerous investigators have made systematic radio measurements of the Solar Corona in terms of its effect on the signal from radio stars which happen to pass close to the sun. The most common type of measurement is in terms of the amplitude of the interference fringes observed on a short baseline interferometer. This can be shown to be equivalent to a measurement of the angular width of the scattered radiation and a relatively simple theory is available for relating this to physical factors. However, these measurements alone do not provide a sufficient basis for inferring absolute phase scintillations.

More recently, Hewish (1964) has reported amplitude scintillation measurements observed on known small diameter radio stars.[6] In conjunction with the previous angular scatter measurements these do provide a basis for the required analysis.

In March and April 1966 the Mariner IV Spacecraft was occulted by the sun. This provided for the first time a basis for direct observation of a narrow band coherent source scattered by the Solar Corona. Preliminary indications were that the one-way signal was broadened to the extent of about 4 cps.[11, 13] However, this data has not been completely reduced. Variation with solar distance and the exact nature of the spectral data is not yet available[1]. When these data are available they will probably provide the best direct basis for the estimation of this effect.

Charged particles in the interplanetary medium, largely due to solar corona outflow, modify the effective refractive index from that of free space by[10]

$$\eta = \sqrt{1 - \left(\frac{f_N}{f}\right)^2}$$

where $f_N^2 = 80.5\,N$

f_N = plasma frequency, cps

N = electron density per cubic meter.

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1. Conversation with Mr. Richard Goldstein, March, 1967.

However, over most of the range of interest in the present problem $f \gg f_N$ so approximately, very nearly

$$\eta = 1 - \frac{1}{2}\left(\frac{f_N}{f}\right)^2 \qquad 15)$$

Furthermore, attenuation can be safely ignored in the region of interest. Direct measurements by interplanetary probes indicates average densities of the order of $10^7/m^3$ at distances of the order of 1 A.U. from the sun, or $f_N \approx 30$ kc in verification of the approximation, 15). The average electron density is expected to vary with the radius outward from the sun as a power law $r^{-\alpha}$ with α theoretically and experimentally in the range 1 to 3 (Hewish, 1963). [5, 7]

There is abundant theoretical and experimental evidence that the solar plasma is highly irregular, tending to assume a filamentary structure originated by the solar corona and controlled by the solar magnetic field. Under these conditions a radio wave passing near the sun may be scattered resulting in random phase, amplitude, and direction of arrival fluctuations at the surface of the earth.

The results of Feinstein (1954) [2], Bramley (1951) [1] and Fejer (1953) [3] show that even though the pertinent plasma volume may be deep, the scattering effect at any point outside the plasma is equivalent to that of a plane phase-changing diffracting screen which introduces a random phase change $\Phi(x,y,t)$. The statistics of this equivalent phase changing screen may be characterized by

- a phase variance

$$\Phi_m^2 = \overline{\Phi^2}$$

- a spatial autocorrelation function

$$\rho_\Phi(\mu) = \frac{\overline{[\Phi(x,t)\,\Phi(x+\mu,t)]}}{\Phi_m^2}$$

- an amplitude distribution

$p(\Phi)$.

For a single blob of size L and electron density excess ΔN, the effective path length error is

$$\Delta R = L_{(m)} \Delta \eta$$

$$= L_{(m)} \cdot \frac{40.25}{f^2_{(sec^{-1})}} \Delta N_{(m^{-3})}$$

and the corresponding phase error is

$$\Delta \Phi_L = \frac{2\pi \Delta R}{\lambda}$$

$$= \frac{2\pi \; 40.25 \; L \; \Delta N}{c \quad f}$$

$$= (8.4)\, 10^{-7} \frac{L_{(m)} \Delta N_{(m^{-3})}}{f_{(sec^{-1})}} \qquad 16)$$

$$\begin{pmatrix} L \text{ in meters} \\ \Delta N \text{ in meter}^{-3}. \end{pmatrix}$$

In traversing a thickness, z, of the medium, the wave encounters $n = \frac{z}{L}$ blobs of size L, and assuming the total effect to accumulate incoherently, the rms random change of effective length is

$$\Delta \Phi_{z_{rms}} = \left(\frac{z}{L}\right)^{1/2} \Delta \Phi_{L_{rms}}$$

$$\Delta \Phi_{z_{rms}} = (8.4 \cdot 10^{-7}) \frac{\sqrt{ZL}\; \Delta N_{rms}}{f} \qquad 17)$$

or

$$\Phi_m^2 = \Delta \Phi^2_{z_{rms}}$$

$$\boxed{\Phi_m^2 = \frac{(71 \cdot 10^{-10})\, z_{(km)} L_{(km)} \overline{\Delta N^2_{(cm^{-3})}}}{f^2_{(mc)}}} \qquad 18)$$

For analytic convenience, the spatial autocorrelation function has generally been assumed of Gaussian form

$$\rho_{\Phi}(u) = e^{-\frac{u^2}{L^2}} \tag{19}$$

where L = effective scale length of the structure.

The distribution function is also generally assumed normal so that

$$p(\Phi)d\Phi = \frac{1}{\sqrt{2\pi}\,\Phi_m} e^{-\left[\frac{\Phi^2}{2\Phi_m^2}\right]} d\Phi \tag{20}$$

Defining the phase difference at two points,

$$\Delta\Phi(x,u) = \Phi(x) - \Phi(x+u)$$

it follows that this is also normally distributed

$$p(\Delta\Phi, u)d\Delta\Phi = \frac{1}{\sqrt{2\pi}\,\Delta\Phi_m(u)} e^{-\left[\frac{(\Delta\Phi)^2}{2\Delta\Phi_m^2(u)}\right]} \tag{21}$$

where the variance of the phase difference is easily shown to be

$$\Delta\Phi_m^2(u) = 2(1 - \rho_{\Phi}^2(u)) \tag{22}$$

Under these assumptions if a unit plane wave is incident on the screen, the complex covariance function of the field just below the screen is

$$\begin{aligned} r_f(u) &= \overline{[f(x)\, f^*(x+u)]} \\ &= \overline{[e^{2\pi i(\Phi(x) - \Phi(x+u))}]} \\ &= \overline{e^{2\pi i\,\Delta\Phi(x,u)}} \end{aligned}$$

Using the Gaussian distribution, the average can be carried out over the ensemble distribution

$$r_f(u) = \int_0^{\infty} e^{2\pi i \Delta\Phi} p(\Delta\Phi, u)\, d\Delta\Phi$$

$$= e^{-\Phi_m^2 (1 - \rho_\Phi(u))} \qquad 23)$$

$$\boxed{r_f(u) = e^{-\Phi_m^2 (1 - e^{-u^2/L^2})}} \qquad 24)$$

If, as will certainly be the case for high frequencies,

$$\Phi_m^2 \ll 1$$

then expanding

$$r_f(u) = 1 - \Phi_m^2 + \Phi_m^2 e^{-u^2/L^2}$$

At a distance from the diffracting screen, e.g., at the earth, it can be shown (e.g., Ratcliff, 1956,p.213) [1/2] that the complex field correlation function remains the same as that just below the screen so that 24) also describes the field at the earth. It is immediately seen that the total incident power, as might be measured for example by an omnidirectional incoherent detector or photon detector is a non-fluctuating constant. A radio antenna, however, does not work in this way; it performs a phase sensitive or coherent weighted summation of the field components ariving from all different directions. It can be shown (e.g., Ratcliff,1955, Feinstein,1954 for two dimensional screen) that a field characterized by a covariance function such as 24) is equivalent to, i.e., may be described in terms of the summation of components ariving from over a distribution, i.e., a spectrum of angles of arrival. The angular spectrum is given by the Fourier transform of the covariance function 24) expressed in natural distance units of wavelength λ, at the carrier frequency. I.e., if θ represents the angle of arrival (more accurately, the sine of the angle of arrival for large θ, but the distinction is academic for our purposes) then

$$G(\theta) = \int_{-\infty}^{\infty} r_f(u) e^{-2\pi i \frac{u}{\lambda}\theta}\, du$$

and inversely

$$r_f(u) = \int_{-\infty}^{\infty} G(\theta) e^{2\pi i \frac{u}{\lambda}\theta}\, d\theta \qquad 25)$$

Thus the angular distribution of the arriving power is seen to have a direct interpretation as the wave number spectrum of the distribution of phase inhomogeneity across the screen, expressed in terms of wavelength as the natural unit of length, i.e., wavenumber

$$k = \frac{2\pi\theta}{\lambda}$$

This forms a direct basis for estimating the spatial distribution of phase inhomogeneity at the screen in terms of angular distribution measurements at the ground.

The complete measurement of the angle of arrival distribution or spectrum will generally include a discrete or delta function component at $\theta = 0$ corresponding to the unscattered power and a diffuse scattered component. In fact, under the assumptions stated previously, if $r_f(u)$ is of the form

$$r_f(u) = e^{-\Phi_m^2 (1 - \rho_\Phi(u))} \qquad 26)$$

with

$$\rho_\Phi(u) = e^{-u^2/L^2} \qquad 27)$$

Bramley (1955) has shown that in general the angular spectrum is

$$G(\theta) = e^{-\Phi_m^2} \left[\delta(\theta) + \pi^{1/2} \frac{L}{\lambda} \sum_{n=1}^{\infty} \frac{\Phi_m^{2n}}{n!n^{1/2}} e^{-\left(\frac{\pi^2 L^2 \theta^2}{\lambda^2 n}\right)} \right] \qquad 28)$$

The unscattered component is given by $s^2 = e^{-\Phi_m^2}$ and the total power in the scattered components is $N_T^2 = 1 - e^{-\Phi_m^2}$. If, as will certainly be the case at high frequencies (above, say, 1000 mc), $\Phi_m^2 \ll 1$ then 26) becomes

$$r_f(u) = 1 - \Phi_m^2 (1 - \rho_\Phi(u))$$

$$= (1 - \Phi_m^2) + \Phi_m^2 e^{-u^2/L} \qquad 29)$$

Also under these conditions, only the first term in the summation is important and the scattered component is given by

$$N^2(\theta) = e^{-\Phi_m^2} \pi^{1/2} \Phi_m^2 \frac{L}{\lambda} e^{-\frac{\pi^2 L^2 \theta^2}{\lambda^2}} \qquad 30)$$

$$\boxed{N^2(\theta) \approx \pi^{1/2} \Phi_m^2 \frac{L}{\lambda} e^{-\frac{\pi^2 L^2 \theta^2}{\lambda^2}}}$$

Conversely, if $\Phi_m^2 \gg 1$ (for low frequencies), then it is clear that in $r_f(u)$ we will be concerned only with the values of $\rho_\Phi(u)$, very nearly unity, i.e., $u/L \ll 1$ and can represent

$$\rho_\Phi(u) = 1 - \frac{u^2}{L^2}$$

giving

$$r_f(u) = e^{-\frac{\Phi_m^2 u^2}{L^2}}$$

similar in shape to the (random part of the) covariance function in the preceeding case but with an effective scale length L/Φ_m, shorter by the factor $1/\Phi_m$. It can be reasoned heuristically that this reduced effective scale length will lead to a correspondingly increased spread of the angular scatter and this has actually been shown by

[4]

Chandrasekhar and by Fejer (Ref. Hewish, 1955) who derive effective angular spreads (e^{-1} scale widths) of

$$\left(\frac{\sqrt{2}}{\pi S/4}\right)\frac{\lambda\Phi_m}{L} \qquad \text{and} \qquad \left(\frac{1}{\pi}\right)\frac{\lambda\Phi_m}{L}$$

respectively, which are almost equal except for small numerical factors.

The choice of the symbols S and N to represent the direct and scattered components is intentional, and meant to suggest that under appropriate conditions of randomness of the phase of the scattered components S and N do indeed take the roles of signal and noise and permit solving for phase, (α), and amplitude, (ΔV), fluctuations directly in terms of the familiar relations

$$\Delta\alpha^2 = \left(\frac{\Delta V}{V}\right)^2 = \frac{N^2}{2S^2} = \frac{1 - e^{-\Phi_m^2}}{2e^{-\Phi_m^2}} \tag{31}$$

The "appropriate conditions of phase randomness" may be shown to correspond to the requirement that the receiver lies in the far zone or Fraunhofer diffraction region relative to the blob size. Clearly in the immediate vicinity of the phase modulating diffracting screen all the noise components are in phase quadrature to the signal producing pure phase and no amplitude modulation. For blobs of size L as one recedes from the screen, beyond a distance of the order of $D = \frac{L^2}{\lambda}$ the phase of the scattered components becomes essentially random and the above conditions are satisfied.

[14]

These considerations may be incorporated in the formalism derived by Wheelon (1957) defining the effective wavenumber filter function

$$F(x) = \frac{x^2}{2(1 + x^2)} \tag{32}$$

Then the effective in-phase noise is given by

$$N_c^2 = \int_{-\infty}^{\infty} N^2(\theta)\, F\left(\frac{2\pi\theta^2 D}{\lambda}\right) d\theta \tag{33}$$

whereas the quadrature noise is

$$N_s^2 = \int_{-\infty}^{\infty} N^2(\theta)\left(1 - F\left(\frac{2\pi\theta^2 D}{\lambda}\right)\right) d\theta$$

$$= N^2 - N_c^2 \qquad 34)$$

Asymptotically

$$F(x) \rightarrow \begin{cases} \frac{1}{2} & x \gg 1 \\ \frac{x^2}{2} & x \ll 1 \end{cases}$$

Under the preceeding assumptions and as discussed, the angular scatter distribution $N^2(\theta)$ is reasonably well represented in any case as of Gaussian form

$$N^2(\theta) = N_T^2 \frac{1}{\sqrt{\pi}\,\epsilon} e^{-\left[\frac{\theta^2}{\epsilon^2}\right]} \qquad 35)$$

$$\text{where} \quad \epsilon = \begin{cases} \frac{\lambda}{\pi L} & (\Phi_m^2 \ll 1) \\ \frac{\lambda \Phi_m}{\pi L} & (\Phi_m^2 \gg 1) \end{cases} \qquad 36)$$

or

$$N_c^2 = N_T^2 \frac{1}{\sqrt{\pi}\,\epsilon} \int_0^{\infty} e^{-\frac{\theta^2}{\epsilon^2}} \frac{\left(\frac{2\pi\theta^2 D}{\lambda}\right)^2}{\left(1 + \left(\frac{2\pi\theta^2 D}{\lambda}\right)^2\right)} d\theta$$

$$= N_T^2 \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-y^2} \frac{2\eta^2 y^2}{1 + 2\eta^2 y^2} dy \qquad 37)$$

$$\text{where} \quad \eta = \frac{\sqrt{2}\,\pi \epsilon^2 D}{\lambda}$$

Thus

$$N_c^2 = \begin{cases} N_T^2/2 & (\eta^2 \gg 1) \\ N_T^2 \eta^2/2 & (\eta^2 \ll 1) \end{cases} \qquad 38)$$

and from 16)

$$N_s^2 = \begin{cases} N_T^2/2 & (\eta^2 \gg 1) \\ N_T^2 (1 - \eta^2/2) & (\eta^2 \ll 1) \end{cases} \qquad 39)$$

$$\text{where } N_T^2 = (1 - e^{-\Phi_m^2})$$

$$\eta = \frac{\sqrt{2}\,\pi \epsilon^2 D}{\lambda}$$

Note that $\eta \gg 1$ corresponds to large distances, large scattering angles, and small wavelength, i.e., the far field or Fraunhoffer diffraction region, with random phasing and equal in-phase and quadrature noise. $\eta \ll 1$ on the otherhand corresponds to the near field where quadrature noise predominates. In any case the AM and FM noise are given by

$$\boxed{\begin{aligned} \overline{\left(\frac{\Delta V}{V}\right)^2} &= \frac{N_c^2}{S^2} \\ \overline{\Delta\alpha^2} &= \frac{N_s^2}{S^2} \end{aligned}} \qquad 40)$$

for the approximation where they are respectively small with respect to 1.

3.1. Analysis of Existing Experimental Data

The preceding results, in particular the equations summarized in the following, permit an analysis and extrapolation of existing experimental results on the terrestrial and interplanetary ionosphere scintillation:

Variation of the basic phase modulation depth with frequency,

$$\Phi_m^2 = (71 \cdot 10^{-10}) \frac{Z_{km} L_{km} \overline{\Delta N^2_{(cm^{-3})}}}{f^2_{(mc)}} \tag{41)}$$

Angular distribution of scatter,

$$N^2(\theta) = \begin{cases} (1 - e^{-\Phi_m^2}) \left[\dfrac{\pi^{1/2} L}{\Phi_m \lambda} e^{-\frac{\pi^2 L^2 \theta^2}{\Phi_m^2 \lambda^2}} \right] & (\Phi_m^2 \gg 1) \\ \Phi_m^2 \left[\dfrac{\pi^{1/2} L}{\lambda} e^{-\frac{\pi^2 L^2 \theta^2}{\lambda^2}} \right] & (\Phi_m^2 \ll 1) \end{cases} \tag{42)}$$

1/e angular width of the scattered radiation,

$$\epsilon = \begin{cases} \dfrac{\lambda \Phi_m}{\pi L} & (\Phi_m^2 \gg 1) \\ \dfrac{\lambda}{\pi L} & (\Phi_m^2 \ll 1) \end{cases} \tag{43)}$$

Unscattered signal power,

$$\begin{aligned} C^2 &= e^{-\Phi_m^2} \\ &\approx 1 - \Phi_m^2 \qquad (\Phi_m^2 \ll 1) \end{aligned} \tag{44)}$$

Total scattered signal power,

$$N_T^2 = 1 - e^{-\Phi_m^2}$$

$$\approx \Phi_m^2 \qquad (\Phi_m^2 \ll 1) \qquad 45)$$

In-phase scattered power,

$$N_c^2 = \begin{cases} N_T^2/2 & (\eta \gg 1) \text{ (Far Field)} \\ N_T^2\,\eta^2/2 & (\eta \ll 1) \text{ (Near Field)} \end{cases} \qquad 46)$$

Quadrature scattered power,

$$N_s^2 = \begin{cases} N_T^2/2 & (\eta \gg 1) \text{ (Far Field)} \\ N_T^2\,(1 - \frac{\eta^2}{2}) & (\eta \ll 1) \text{ (Near Field)} \end{cases} \qquad 47)$$

where $\eta = \sqrt{2}\,\pi\varepsilon^2 D/\lambda$

A.M. Variance,

$$\overline{\left(\frac{\Delta V}{V}\right)^2} = \frac{N_c^2}{S^2} \qquad \left(\overline{\left(\frac{\Delta V}{V}\right)^2} \ll 1\right) \qquad 48)$$

P.M. Variance

$$\overline{\Delta\alpha^2} = \frac{N_s^2}{S^2} \qquad (\overline{\Delta\alpha^2} \ll 1) \qquad 49)$$

Figure 2 is a plot of the scattering angle ε vs radius from the sun combining determinations from a number of sources [5] and at several different wavelengths, reduced to a common reference wavelength of 7.9 meters (38 mc) based on the λ^2 law confirmed in many experiments up to 178 mc. The observations may be described in terms of a power law relationship of the approximate form

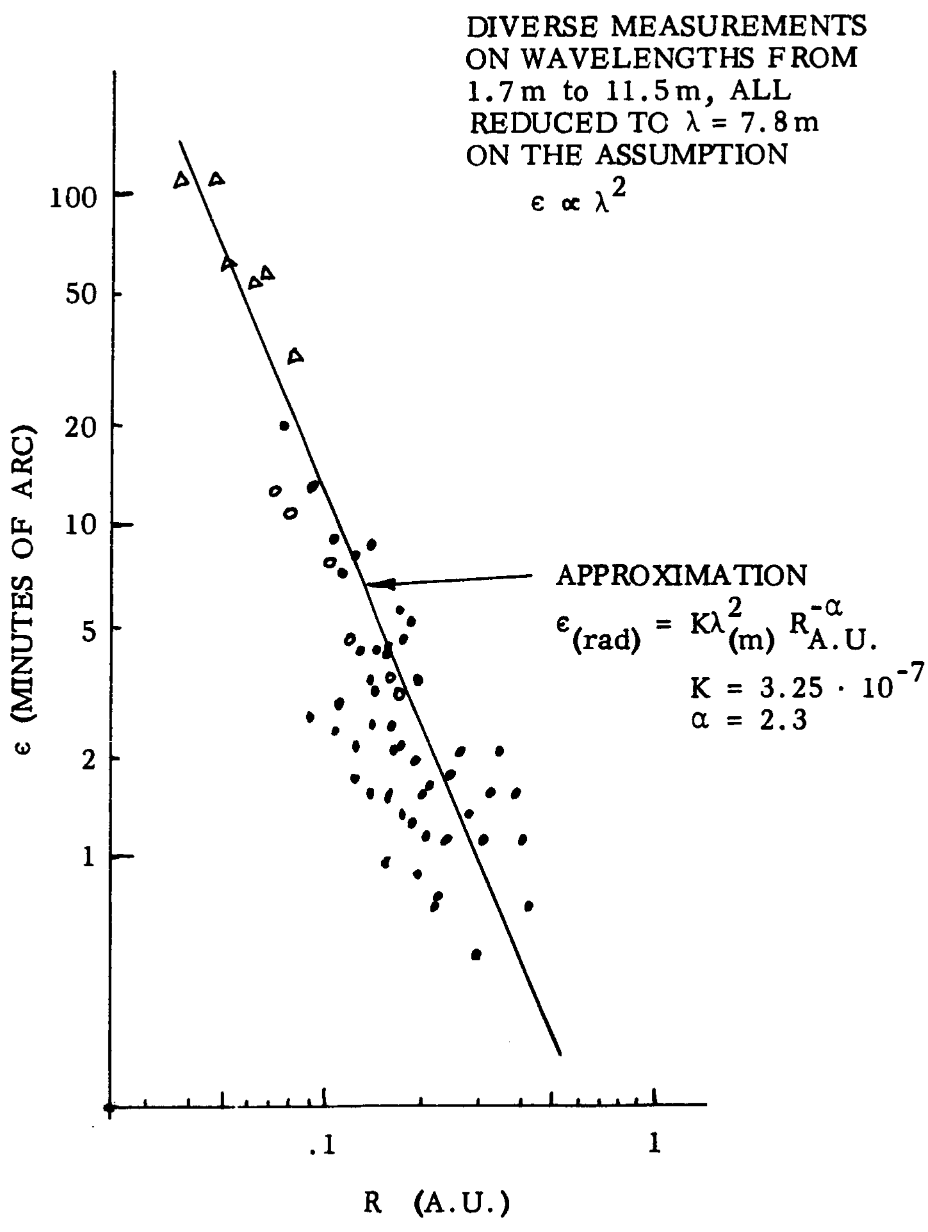

FIGURE 2

SCATTERING ANGLE MEASUREMENTS

$$\epsilon = K\lambda^2_{(m)} \, R^{-\alpha}_{(A.U.)} \tag{50}$$

where $\alpha \approx 2.3$

$$\left.\begin{aligned} K &= 3.25 \cdot 10^{-7} \text{ rad} \\ &= .067 \text{ sec.} \end{aligned}\right\} \quad .05 < R_{(A.U.)} < .5$$

From which we can infer (using the relation for $\Phi_m >> 1$ which may be inferred from the λ^2 law)

$$\epsilon = \frac{\lambda \Phi_m}{\pi L}$$

$$\frac{L_{(m)}}{\Phi_m} = \frac{\lambda}{\pi \epsilon} = \frac{1}{\pi K R^{-\alpha}_{(A.U.)} \lambda_{(m)}} \tag{51}$$

For a particular example pertinent to a later comparison, at $\lambda = 1.7\,m$ (178 mc/s), $R = .5$ A.U.

$$\begin{aligned} \epsilon &= 4.7 \cdot 10^{-6} \text{ rad} \\ &\approx 1 \text{ sec} \end{aligned}$$

$$\frac{L}{\Phi_m} = 1.1 \cdot 10^5 \text{ meters} \tag{52}$$

Thus for $\Phi_m \geqslant 1$

$$L \geqslant 110 \text{ km} \tag{53}$$

Setting $\eta = 1$, for the critical point between near and far field and taking $D = 1$ A.U. $= 1.49 \cdot 10^{11}$m, $R = .5$ A.U., we find for the critical wavelength

$$\lambda_c = \sqrt{2}\, \pi \epsilon^2 D \tag{54}$$

$$\lambda_{c(m)} = \sqrt{2}\, \pi D_{(m)} K^2 \lambda^4_{c(m)} R^{-4.6}_{(A.U.)}$$

or

$$\lambda_{c(m)} = (\sqrt{2}\,\pi D_m K^2 R_{(A.U.)}^{-4.6})^{1/3} \tag{55}$$

$$= .83 \text{ m}$$

or

$$f_c = 360 \text{ Mc/s}$$

independent of L and Φ_m separately. That is, frequencies less than 360 Mc/s as in most of the experiments reported to date are in the near field while frequencies greater than 360 Mc/s would be in the far field with fully developed amplitude scintillation. This is for R = .5 A.U.. For other R, λ_c scales as $R^{1.53}$,

$$\lambda_{c(m)} = 2.44\, R_{(A.U.)}^{1.53} \tag{56}$$

[6]

Hewish, Scott and Wills (1964) report a series of measurements of amplitude scintillation at 178 Mc/s (λ = 1.7 m) in which amplitude scintillation of the order of 50% was observed for small radio stars ($\lesssim$ .3 sec angular diameter). The observed rate of fluctuation was typically 10 to 30 maxima per minute. For the assumed Gaussian form correlation function of scale length L = 110 km (eq. 53) and drift velocity, v, the resulting fluctuations have a frequency spectrum of the form

$$G(f) \alpha\, e^{-\frac{\pi^2 f^2 L^2}{v^2}} \tag{57}$$

[8]

for which the expected rate of maxima is (Ref. Rice)

$$n = \left[\frac{\int_0^\infty f^4 G(f)\, df}{\int f^2\, G(f)\, df}\right]^{1/2} \tag{58}$$

$$= \sqrt{\frac{3}{2}}\ \frac{v}{\pi L}$$

or for the observed rate

$$v \approx \sqrt{\frac{2}{3}}\ \pi\,(110 \text{ km}) \cdot \frac{1}{3} \tag{59}$$

$$= 94 \text{ Km/sec}$$

which is in reasonable accord with available physical theories. Note that under these conditions the spectral bandwidth to the 3 db point is

$$B = .68\, n$$

or about 0.3 cps.

Hewish's amplitude fluctuation data are replotted in Figure 3 from which it can be seen that there is a strong variation with R, the radial distance of point of closest approach of the line-of-sight to the sun which may be expressed in the approximate form

$$m = \sqrt{\frac{\overline{\Delta V^2}}{V}}$$

$$\approx .07\, R_{(A.U.)}^{-2.3} \qquad (.3 < R_{(A.U.)} < .9) \qquad (\lambda = 1.7 \text{ m}) \qquad 60)$$

whence from 48) we can infer the "in phase" noise-to-signal ratio

$$\frac{N_c^2}{S^2} = m^2 \qquad 61)$$

In order to relate this to the total noise-to-signal ratio we need the parameter η which from 54) and 56) may be written

$$\eta = \lambda_c/\lambda \qquad 62)$$

$$= 1.43\, R_{(A.U.)}^{1.53} \qquad (\lambda = 1.7 \text{ m}) \qquad 63)$$

Note that this is less than 1 for $R < .8$ A.U. so that over this range we may use the small η approximation

$$N_c^2 = N_T^2\, \eta^2/2$$

and

$$\frac{N_T^2}{S^2} = \frac{2m^2}{\eta^2}$$

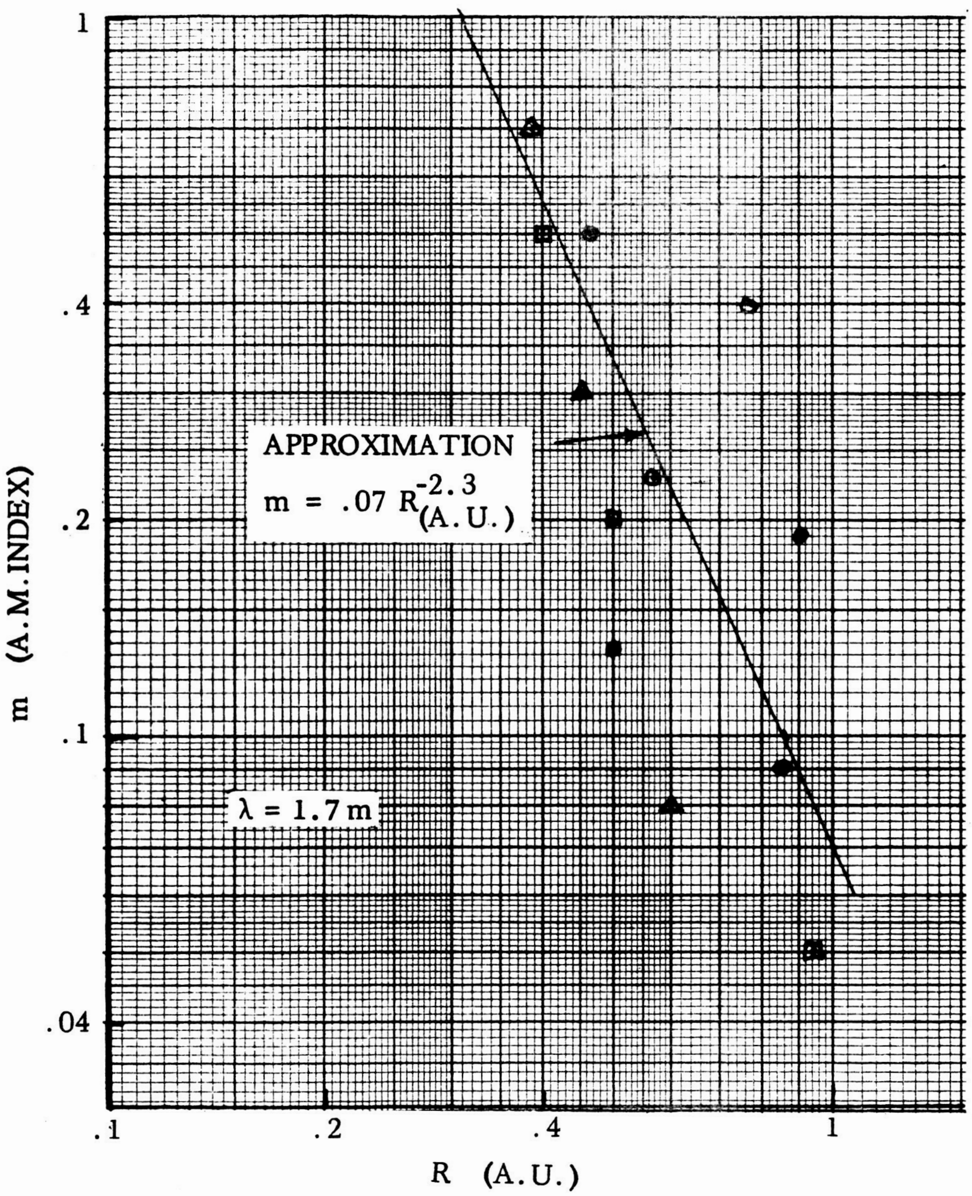

FIGURE 3
MEASURED AMPLITUDE SCINTILLATION

$$\frac{N_T^2}{S^2} = .047\, R_{(A.U.)}^{-7.66} \qquad 64)$$

$$= \frac{1 - e^{-\Phi_m^2}}{e^{-\Phi_m^2}}$$

From which

$$\Phi_m^2 = \log(1 + .0047\, R_{(A.U.)}^{-7.66})$$

$$= \log\left(1 + \left(\frac{R_{(A.U.)}}{0.5}\right)^{-7.66}\right) \qquad (\lambda = 1.7\ m) \quad (.3 < R_{(A.U.)} < .8) \qquad 65)$$

It is to be emphasized that there is no theoretical justification for the form of this equation, it is simply the natural result of a fit to experimental data over a very limited range of parameter.

At other wavelengths, by 41), Φ_m^2 scales as λ^2

$$\Phi_m^2 = \left(\frac{\lambda}{1.7}\right)^2 \log\left(1 + \left(\frac{R_{(A.U.)}}{0.5}\right)^{-7.66}\right) \qquad 66)$$

The resulting phase fluctuation as given by eqtns. 44,45,47, and 49 is plotted in Figure 4 as a function of R for several wavelengths.

At wavelengths of less than 0.3 meters (1000 Mc) and R greater than 0.3 A.U. the total rms phase fluctuation is well under 0.5 radian and would not seriously deteriorate phase-locked tracking performance regardless of the rate of fluctuation. For closer approaches to the sun, adequate measurements do not appear to be available other than by gross extrapolation of the present results. The various available physical theories of radial variation are of such diversity in their predictions as to be of little help in this regard.

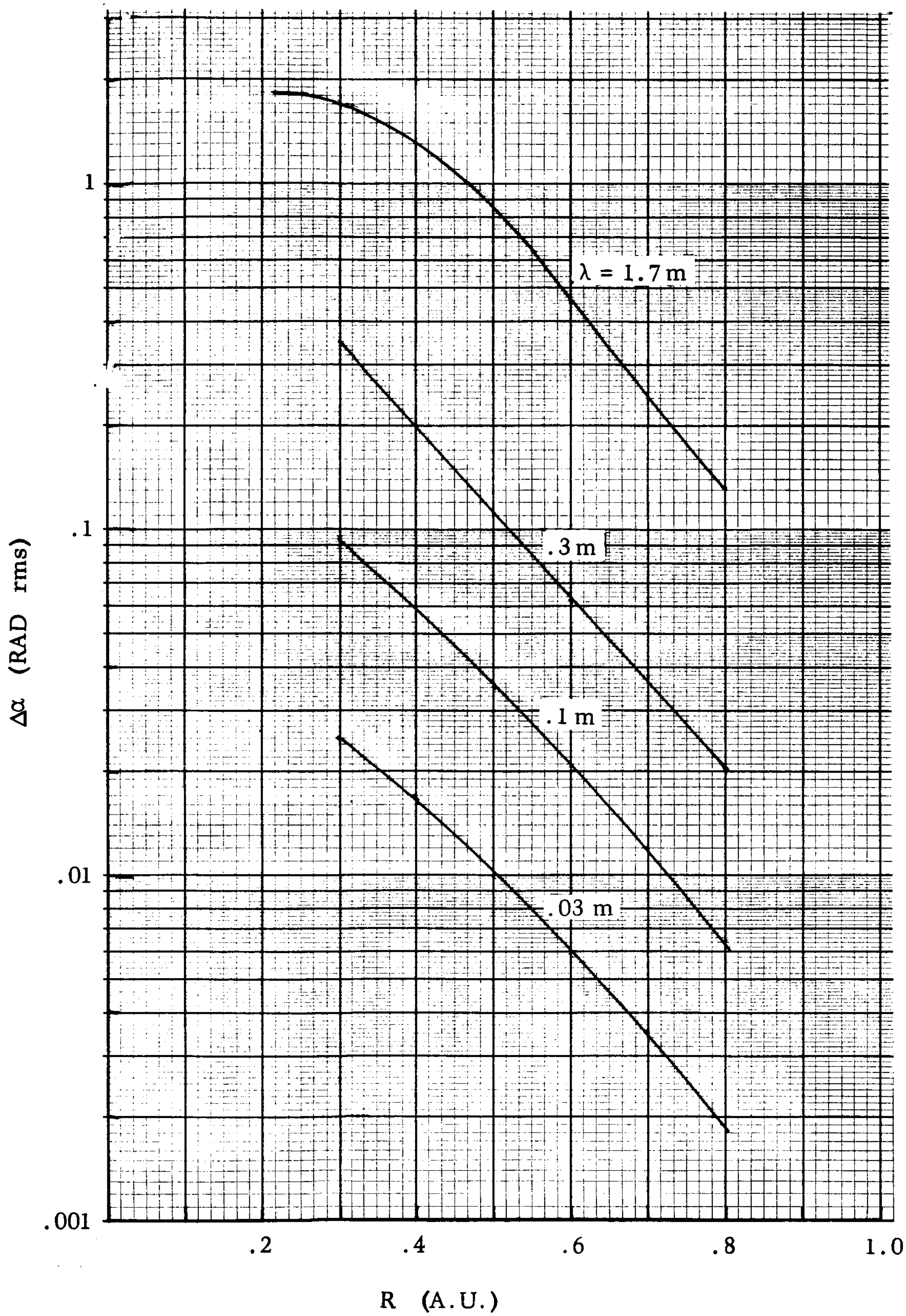

FIGURE 4

PREDICTED PHASE FLUCTUATION

Even in cases where the total phase fluctuation would exceed, say, 1 radian, the spectrum of the phase fluctuations should be the same as that for the observed amplitude fluctuations, i.e., about 0.3 cps, so that only very narrow band tracking loops would be adversely affected.

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